Product Development with Lurking Patentees*

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Abstract

We analyze technology investment and adoption by a product developer who faces uncertainty over whether any existing patent covers a new technology. If there is a patent, the (non-producing) patentee may choose to "lurk"—i.e., to strategically delay enforcement—hoping that the developer will adopt the patented technology and unwittingly accumulate infringement liability. Lurking incentives are pervasive and strongest at intermediate levels of patent strength and damages. We identify when there is inefficiency relating to adoption or investment and find that lurking does not usually deter developers from efficient technology adoption. Ignoring the impact of lurking on the developer's strategic choices about technology adoption and investment can misguide economic predictions and policy conclusions.

JEL Classification: K2, O3.

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1 Introduction

When considering the adoption of technology, product developers are frequently uncertain about whether they might infringe any existing patents. The rise in the number of patents over recent decades has created a significant infringement risk for developers. Thus, when choosing which technology to adopt, a developer needs to consider both the commercial value and the infringement risk associated with each option. However, even if the developer cannot identify any relevant patents ex ante (before adopting a technology), the owner of such a patent (if one exists) may be able to identify the developer as a potential licensee. In that event, the patentee may approach the developer early on to initiate licensing negotiations before the developer adopts a technology. At first blush, this would seem to benefit both sides: the developer benefits from an early resolution of uncertainty, and the patentee avoids the risk that the developer will adopt an alternative non-infringing technology.

However, the patentee may instead prefer to "lurk"—to delay enforcement of the patent strategically. In such a case, the patentee hopes that the developer will unwittingly adopt an infringing technology, leading it to accumulate infringement damages with every sale it makes. Because patent damages awards are highly imprecise, they will often exceed the incremental value of the patented technology. In such a case, the patentee has incentives to "hold up" the developer—to extract higher license fees ex post than the developer would have willingly paid ex ante. A forward-looking developer will factor the risk of lurking patentees into its technology adoption and R&D decisions.

Consider the context of technology standard-setting.[1] Product developers deliberate over ways to implement different candidate standards, each subsuming many complex technologies. But the extent to which existing patents cover these technologies is uncertain. In some cases, patent owners may approach the developers early on to claim patents covering parts of a candidate standard. However, developers cannot assume that there are no applicable patents if patentees do not approach them. There is also a risk that someone owns a relevant patent but strategically waits to assert it until after product developers have committed to it. Although this kind of behavior is often discussed in the standard-setting context (e.g. Farrell et al., 2007), it has the potential to occur in any situation where innovative product developers have incomplete information about relevant patents.

To explore these strategic issues, we study a dynamic game of incomplete information with an (ex ante) unknown number of players. In the baseline game, a developer chooses between

[1] At the end of the Introduction, we provide additional illustrative cases.

a safe technology and a risky one. The risky technology yields higher profits and social value, but there is some probability it will infringe an existing patent. The uncertain number of players is a consequence of this incomplete information about patents: the developer does not know if there is any patentee whose patent covers the risky technology. If there is, then the (non-practicing) patentee may approach the developer for an ex ante license, or it may lurk hoping for a chance to sue for infringement (to obtain an ex post license) if the developer chooses the risky technology. If there is no patentee, the developer's technology choice will not be opposed by anyone.

Technology adoption and the timing of patent enforcement are endogenously determined in equilibrium, as are the developer's preceding decisions about R&D investment. If a patentee does not approach, the developer must adopt a technology under uncertainty about whether there is a lurking patentee or there are no applicable patents. The developer's beliefs about whether there is a lurking patentee play a central role in our analysis and shape most of the strategic behavior.

Many authors have suggested that the risk of patent holdup could lead developers to make inefficient technology adoption or investment decisions (see, e.g., Lemley and Shapiro, 2007; Reitzig et al., 2007; Lemley and Melamed, 2013). However, models in the existing literature omit most of the strategic behavior that occurs before holdup can potentially take place. Specifically, the literature focuses on developers who have already committed to an infringing technology and are subsequently "surprised" when a patent is enforced against them ex post (see, e.g., Lemley and Shapiro, 2007). But a rational, forward-looking developer will account for the risk of lurking patentees when choosing a technology or investing in R&D, influencing the patentee's incentives to lurk. Moreover, the developer's incomplete information about patents plays a critical role in shaping strategic behavior. Indeed, when developers are forward-looking, lurking has strategic value only if developers have incomplete information.

Our model contributes qualitatively new results and insights. Specifically, we show that lurking, ex post enforcement, and developer regret occur in equilibrium even though developers have alternative technology choices and act strategically. Intuitively, the developer discounts the infringement risk by the probability of a patentee, so it takes risks in many situations when there is, in fact, a patentee. Moreover, the developer finds these infringement risks acceptable even in cases where it knows that patentees never approach for ex ante licenses *and* where the expected royalty (conditional on an ex post enforcement) exceeds the incremental contribution of the patented technology. For the patentee, lurking in such situations yields an expected royalty payoff that is, due to better bargaining leverage, strictly superior to that in ex ante licensing. In equilibrium, the patentee earns a relatively high payoff and the developer earns a relatively low payoff, but technology adoption is efficient. Ex post enforcement is consistent with empirical evidence suggesting that NPEs tend to delay patent enforcement (e.g. Love, 2012; Cohen et al., 2019).[2]

We also find that fear of lurking patentees can lead to inefficient adoption but this effect cannot become too widespread. For one thing, the developer always adopts the risky technology if the expected royalty is "low" (i.e., below that technology's incremental contribution), or "intermediate" (i.e., slightly above that increment), as such adoption carries a positive but acceptable risk. For another, if the expected royalty is "high" enough that infringement risks are unacceptable, lurking becomes risky for the patentee. The developer strategically responds to high infringement risk by sometimes adopting the safer (but less valuable) technology, reducing the strategic value of lurking. These forces reduce the equilibrium probability of efficient technology adoption. But interestingly, this probability varies non-monotonically with a higher expected royalty. In the range where lurking is risky, patentees approach more often as the expected royalty increases. This yields more ex ante licenses.

We also study the impact of patentee lurking on ex ante R&D investment by the developer. When the prospect of a lurking patentee reduces the value of discovering a new technology, the developer underinvests in R&D. Interestingly, with a low expected royalty, lurking has no impact on the developer's behavior. The reason is that the developer finds it worth investing in R&D even when infringement is certain. For an intermediate expected royalty, lurking leads the developer to underinvest in R&D but to adopt the best available technology. Here, lurking creates a moderate infringement risk, which the developer is willing to accept by adopting the efficient technology . However, even a moderate infringement risk reduces the developer's expected payoff, hindering R&D investment. When the expected royalty is high, the fear of infringement pushes the developer to adopt the safer technology some times, which means that lurking leads to inefficiency in both investment and adoption.

We extend our model to accommodate for the possibility of costly search. First, we allow the developer to do a "freedom to operate" search such that the developer pays a cost to assess ex ante whether the risky technology may infringe any existing patents. We show that the developer's willingness to pay for a freedom-to-operate search is non-monotone and has a single peak at an intermediate value of the expected royalty. For a low expected royalty, a freedom-to-operate search has no value, since adopting the potentially infringing

[2] A non-practicing entity (pejoratively called a "patent troll") is a firm that owns and enforces patents but does not itself use its patents to manufacture products. See, e.g., the 2016 FTC study on Patent Assertion Entity Activity (page 1).

technology is strictly dominant. For an intermediate expected royalty, the developer adopts the risky technology but a larger expected royalty increases the risk, increasing the value of a freedom-to-operate search. For a high expected royalty, a freedom-to-operate search is less valuable since the patentee sometimes approaches to avoid the developer's adoption of the safe technology. This implies that the developer may forgo the option to learn about potential patentees before adopting a technology when the expected royalty is either low enough or high enough. These results provide an economic basis for prior work (e.g., Lemley (2008); Chien (2014)), noting that firms in innovative industries often "ignore patents" when launching new products.

Second, we allow the patentee to engage in a costly search to identify developers who might infringe on its patent. This differs from the baseline game, which assumes the patentee can identify the developer for free. If it does not search (or if the search fails), it cannot identify the developer unless it adopts an infringing technology; this makes ex ante licensing impossible. In this setting, the patentee rationally does not search unless the expected royalty is high, since that is the only case where the developer may not adopt the infringing technology. Interestingly, these two results combined show that when the expected royalty is low both developers and patentees have no incentive to search for each other ex ante, despite both recognizing that they may come into conflict ex post.

We also study the impact of allowing the developer to file a declaratory judgment action (a preemptive lawsuit to address patent infringement) if ex ante licensing negotiations break down. This may help the developer, who benefits by having the chance to resolve uncertainty about the royalties before adopting a technology. We show that a declaratory judgment helps precisely in situations where a court-awarded royalty would be high but the developer believes that infringement is unlikely because the patent is weak, so he chooses the risky technology anyways. However, the developer can only bring a declaratory judgment action if he knows the patentee's identity, which requires that the patentee approaches him ex ante. Since declaratory judgments may make ex ante licensing worse for the patentee, it yields more lurking in equilibrium.

We also relax the assumption of costless litigation, which introduces two new effects. First, when the expected royalty is extremely low, the patentee does not have a credible litigation threat, so the developer faces no risks. Second, if litigation is credible, the parties save on litigation costs when they agree on a license, which creates an additional surplus to bargain over. Depending upon litigation costs and bargaining ability, either party may earn more as a result of this sharing. We say that the party that benefits obtains a litigation-cost advantage, and show that patentee is more willing to lurk precisely when it has a litigation-cost advantage. And if the litigation-cost advantage exceeds the patented technology's incremental contribution, then the patentee always lurks even for low expected patent damages.

Lastly, we investigate the role of allowing the developer to commit to adopt the safe technology if not approached. This strategy removes all the ex post infringement risk for the developer, making him better off. However, commitment does not restore efficiency because when there is no patentee the developer will never be approached and will inefficiently adopt the safe technology. Furthermore, we show that under commitment the developer inefficiently adopts the safer technology more frequently than in equilibrium when the expected royalty is high.

Related Literature. Our paper is partially related to patent holdup. There is a rich literature on this subject.[3] Patent holdup is often discussed in the context of standard setting (e.g. Farrell et al., 2007), although it can occur much more broadly. The basic concern in this literature is that, if a patent is not enforced until after a product developer has unwittingly committed to an infringing technology, then the patent owner can extract excessive royalties. Models of patent holdup (e.g. Lemley and Shapiro, 2007; Shapiro, 2010) focus on explaining how various factors often lead royalties to be larger if negotiated ex post rather than ex ante.[4]

These papers help explain the economic and legal factors that can facilitate holdup. But by focusing on a developer who has already committed to an infringing technology at the outset, they omit much of the strategic behavior that occurs before a potential holdup problem can arise. Thus, they do not explain when strategic lurking will actually occur in equilibrium when developers are rational, forward-looking agents. Nor do they typically consider R&D investment and endogenous technology adoption by the developer. A partial exception is Reitzig et al. (2007), which also models technology adoption with uncertain infringement. In contrast to our model, its focus is on the developers' entry decision conditional on its beliefs about the chance of infringement. In particular, they do not consider a patentee's ability to choose between ex ante licensing and strategic lurking, which is our central focus.

Our paper also relates to the literature on NPEs.[5] Lemley and Melamed (2013) discuss the increase in the number of patent infringement lawsuits by NPEs, and contrast the behavior of NPEs and practicing entities. The article mentions that NPEs appear more likely to conceal their patent holdings and are "more able to defer licensing discussions until technology users

[3]For a discussion of the literature, see, e.g., Cotter et al. (2019).

[4]Some drivers are technology-specific sunk costs; path dependence; patent remedies that may overcompensate patent owners; and (in the standard setting context) network effects.

[5]See, e.g., Chien (2013); Love et al. (2016).

have developed and invested in products that include the patented technologies and are thus less able to switch to alternatives."[6] Orr (2013) also discusses the value of secrecy for NPEs noting that they may "benefit from concealing their patent interests until after a technology has been accepted and locked in within an industry."[7]

NPEs usually enforce their patents ex post. This has frustrated product developers, led to costly litigation, promoted policy reforms such as the America Invents Act, and motivated disclosure policies of standard-setting organizations (SSOs).[8] Many authors have suggested that the risk of patent holdup could lead developers to make inefficient technology adoption or investment decisions (see, e.g., Lemley and Shapiro, 2007; Reitzig et al., 2007; Lemley and Melamed, 2013).

Our results also provide a rationale for developer firms adopting a wait-and-see approach by "ignoring patents." Lemley (2008) discusses at length that developers often adopt a technology without searching for patents that they may infringe. Our paper also relates to the literature on "patent thickets" (Shapiro, 2000). This addresses the concern that it is hard for product developers to introduce new products without potentially infringing numerous broad patents (patents that perhaps should not have been granted in the first place).

More broadly, our paper contributes to the literature on bargaining with private information. Shavell (1989) studies the incentives to reveal information before litigation. Chung et al. (2016) and Eső and Wallace (2019) study the timing of information disclosure during bargaining. Other papers study investments before trading. For instance, in Lau (2008), a seller observes a buyer's investment with some probability before making an offer. In Condorelli and Szentes (2020), the buyer chooses the distribution from which she privately draws her valuation, while the seller only knows the distribution before making an offer.

Examples. Some of the most celebrated inventions in history were put into practice by developers who licensed patents from the inventor. Laszlo Biro invented and patented a commercially-viable ballpoint pen in 1938. The British Royal Air Force licensed this technology during World War II, ushering in the era of the "biro" pen (Ferasso et al., 2017). More recently, offices of technology transfer at various universities have worked with inventors to find suitable licensees. For instance, gene-splicing patents invented by Herbert Boyer and Stanley Cohen in the 1970s helped form the foundation for the modern biotechnology industry. These patents yielded 468 licensees and $254 million in revenue for Stanford University

[6]Lemley and Melamed (2013, p. 2165).

[7]Orr (2013, pp. 543-44).

[8]Significant parts of the literature on patent litigation in the last 20 years have focused on costs stemming from unforeseen lawsuits by NPEs (Bessen et al., 2011; Bessen and Meurer, 2013; Bessen et al., 2018).

through 1997 (Feldman et al., 2007).

In these and other instances, technology inventors and patentees that approach developers for possible licensees do so with better information than the developer about that technology and with some uncertainty about whether the developer may use a particular technology. In some instances, developers may know little about the technology or patent status prior to the approach. For example, Samuel Gart invented an ergonomically-shaped computer mouse in the late 1980s. In 1989, before the patent's issuance, Gart approached Logitech to discuss a possible license for the patent application. Logitech declined, though a separate company (Moustrak) later took a license. Gart's expectation that Logitech might use his technology proved prescient, however; in 1997, he sued Logitech in California, alleging infringement by the TRACKMAN VISTA and TRACKMAN MARBLE products.[9]

Other cases reveal that patentees lurked when they knew developers were likely to use the patented technology. In a notable case, Rambus, Inc. participated in the 1990s in the Joint Electron Device Engineering Counsel (JEDEC), a standard-setting organization (SSO) that developed the SDRAM and DDR SDRAM computer memory standards. Importantly, Rambus did not disclose patent applications relevant to the candidate standards. After manufacturers adopted the standards, Rambus asserted its patents against them. More than a decade of litigation ensued (Besen and Levinson, 2012).

One extreme form of lurking is termed "submarine patenting." In this strategy, the inventor uses continuations to delay a patent's issuance, which keeps the patent "off the books" and potentially leads unwitting infringers to accumulate damages for many years. For example, Jerome Lemelson filed numerous patent applications for machine-vision/bar-code technology in the 1950s, but used continuations to delay the issue of the patents until decades later. In the 1980s and 1990s, Lemelson sued automobile manufacturers and earned hundreds of millions of dollars in settlements (Hansen, 2004). Recent reforms to the patent system may have curbed submarine patenting.[10]

[9] *Samuel Gart v. Logitech, Inc.* 254 F.3d 1334 (Fed. Cir. 2001).

[10] The Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) in 1995 changed the patent term from 17 years after the patent issues to 20 years after the initial patent application (Lemus and Marshall, 2018). Also, the American Inventors Protection Act of 1999 required U.S. patent applications to be published 18 months after their filing date rather than at patent grant (Hegde and Luo, 2018).

2 Model

We present a simplified framework to isolate and highlight the strategic delay in patent enforcement by a non-producing entity. In the subsequent sections, we incorporate additional elements, such as ex ante R&D, search, declaratory judgments, commitment, and costly litigation.

There are either one or two players. There is always a product developer. With probability λ, there is a patentee (an NPE), and with probability $1-\lambda$ there is no patentee. Figure 1 illustrates the timing of the game, the players' actions, and their payoffs. Incomplete information plays a key role and is captured by Nature moving first. The patentee, if there is one, knows about the developer and moves next, choosing one of two options. First, it can *approach* the developer to bargain over a license before the developer has committed to a technology (ex ante licensing). Alternatively, the patentee can *lurk* and delay enforcement, hoping that the developer commits to a technology that infringes on its patents.[11] The developer moves next, making the irreversible choice of which technology, A or B, to adopt and commercialize. Crucially, the developer *does not know* whether there is a patentee. In fact, the developer knows that with ex ante probability $1-\lambda$ there is no patentee. Given that the patentee reveals itself by approaching the developer for an ex ante license, the developer updates its belief that there is a lurking patentee from λ to $\hat{\lambda}$ when not approached, and uses this belief when deciding which technology to adopt.

Technology A is in the public domain, so there is no infringement risk by using it. The developer earns a profit of $(1-\bar{\rho})\pi$ from adopting it, where $\bar{\rho} \in (0,1)$. The developer earns a profit of π from choosing technology B. The parameter $\bar{\rho}$ measures B's incremental value relative to A, $\bar{\rho}\pi$. When $\bar{\rho}$ is low, the developer obtains almost the same payoff from choosing A or B, whereas when $\bar{\rho}$ is high, the developer gets a higher payoff from choosing B. However, technology B carries some risk for the developer because there is chance that a patentee will claim patent infringement. Both technologies cost the same to adopt, and this cost is normalized to zero. Thus, since B is strictly more valuable than A, the ex post infringement risk is the only reason why the developer might pick A.

If the patentee lurks and the developer chooses A, the game ends; the developer gets $(1-\bar{\rho})\pi$ and the patentee gets nothing. If the patentee lurks and the developer chooses B, then, after the adoption, the patentee will bring an infringement lawsuit and demand compensation.

[11] Sections 3.1 and 3.2 consider the possibility that the patentee and/or the developer may search to learn about their counterpart player.

The patentee prevails in litigation with probability θ, which can be viewed as a measure of patent strength. If the patentee wins in litigation, it gets damages of $r\pi$, where $r \in [0,1]$ is the royalty rate set by the court. It is convenient to work with the *expected* royalties $\rho \equiv \theta r$ that the patentee would obtain in litigation. The patentee cannot get an injunction.[12] An alternative to litigation is to settle the lawsuit. In that case, we assume that the license fee is set via Nash bargaining under complete information, where $\beta \in [0,1]$ is the patentee's bargaining ability. Litigation costs are normalized to zero, so the developer pays the patentee the expected royaly $\rho\pi$.[13] Finally, if there is no patentee, the developer earns a profit of π unencumbered from choosing B.

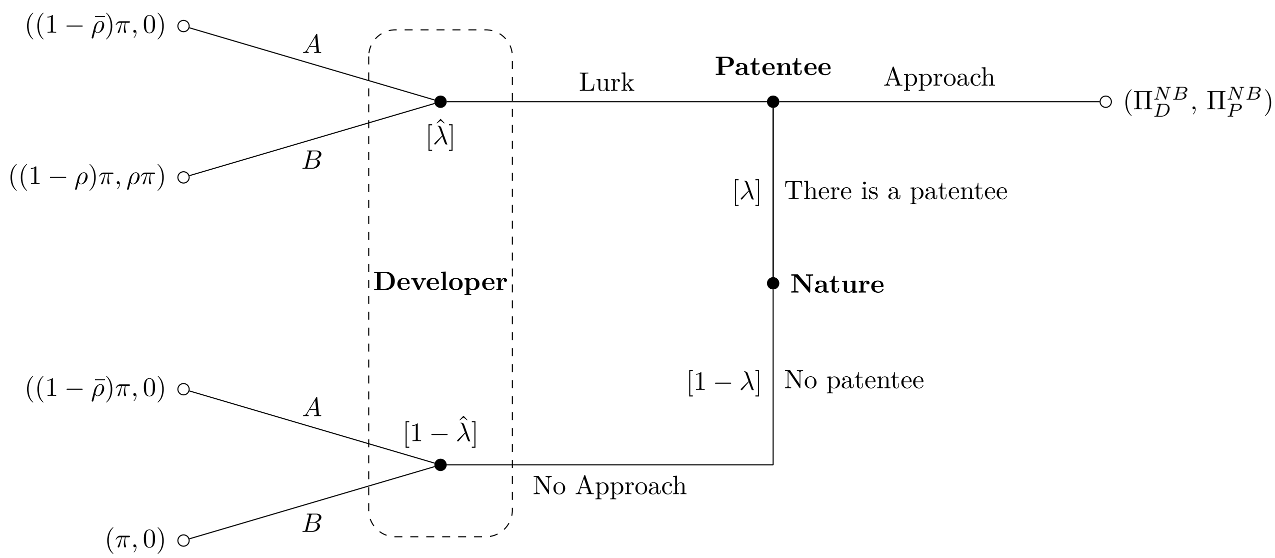

Figure 1: The Baseline Game

When the patentee approaches the developer for ex ante licensing, the license fee is set via Nash bargaining. It is efficient to always adopt B, so the parties will bargain on the price of a (lump-sum) license for this technology. The developer's ex-ante bargaining leverage depends critically on whether adopting A is a credible threat. That is, whether the developer's expected payoff of adopting B and enduring an infringement lawsuit post adoption, $(1-\rho)\pi$, is lower than the developer's payoff of adopting A, $(1-\bar{\rho})\pi$. Our first result identifies the payoffs that result from this Nash bargain, which we denote Π_P^{NB} and Π_D^{NB} for the patentee and developer, respectively.

[12]Following the US Supreme Court's 2006 *eBay* decision, non-practicing patentees typically cannot get injunctions for patent infringement.

[13]We consider costly litigation in Section 3.4

Lemma 1. *If technology* B *is patented and the patentee approaches ex ante, Nash bargaining results in the following payoffs:*

$$\Pi_P^{NB} = \begin{cases} \rho\pi & \text{if } \rho \leq \bar{\rho} \\ \beta\bar{\rho}\pi & \text{if } \rho > \bar{\rho} \end{cases}, \quad \Pi_D^{NB} = \begin{cases} (1-\rho)\pi & \text{if } \rho \leq \bar{\rho} \\ (1-\beta\bar{\rho})\pi & \text{if } \rho > \bar{\rho} \end{cases}.$$

When $\rho \leq \bar{\rho}$, the developer will adopt B even if it is certain to face an infringement claim, because the expected ex post royalties ($\rho\pi$) are lower than the incremental value of B ($\bar{\rho}\pi$). In this case, if the patentee approaches, it is not credible for the developer to threaten to adopt A if bargaining fails. Even if bargaining breaks down, the developer will still end up using B and paying $\rho\pi$ for it (in damages). Thus, in ex ante licensing, they will just agree on that same price of $\rho\pi$, since there is no surplus to bargain over in this case. Otherwise, when $\rho > \bar{\rho}$, the developer can credibly threaten to adopt A if the negotiations fail. Since B is more valuable than A, this means that there is a bargaining surplus (equal to $\bar{\rho}\pi$) that the firms bargain over.

The price of the ex ante license depends on just the patentee's bargaining ability and the incremental value of B relative to A. Importantly, whenever the patentee approaches, the developer ends up adopting B. It is only when the patentee *does not* approach that there is the question of which technology the developer will pick in equilibrium.

Benchmark: A certain threat. Suppose that the developer knows there is a patentee, i.e., $\lambda = 1$. Regardless of whether he knows the identity of the patentee, if he is not approached, he knows *for sure* that the patentee will sue him for infringement after adopting B. This case is very similar to the "early negotiations" model of Lemley and Shapiro (2007), where our developer's choice of A is analogous to their downstream firm's option to redesign. We have the following result:

Lemma 2. *If $\lambda = 1$, the developer chooses B for sure in equilibrium. For $\rho \leq \bar{\rho}$ there are multiple payoff-equivalent equilibria: the patentee approaches with probability $\phi^* \in [0, 1]$ and receives $\rho\pi$.*[14] *For $\rho > \bar{\rho}$, the patentee approaches for sure to negotiate a license ex ante.*

[14] In the Appendix, Section A, equilibrium multiplicity vanishes if the patentee can confer some added value to the developer by approaching early.

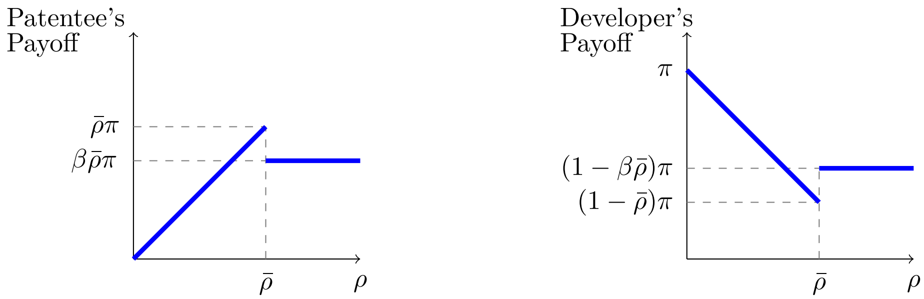

Figure 2: Payoffs for the developer and patentee for the case of a certain patentee ($\lambda = 1$).

When the developer is certain about the existence of a patentee, there is no strategic benefit to lurking. The developer has no doubt that, if not approached, choosing B will eventually trigger an infringement lawsuit. Figure 2 shows the equilibrium payoffs in this case, which coincide with the Nash bargaining payoffs in Lemma 1. Both the patentee's and the developer's payoffs are non-monotonic in ρ and discontinuous at $\rho = \bar{\rho}$ when $\beta < 1$; they are monotonic and continuous when $\beta = 1$. When $\beta < 1$, the patentee does not capture all the surplus created by B when the developer can credibly threaten to adopt A.

If $\rho \leq \bar{\rho}$, payoffs fall with ρ for the developer and rise for the patentee because the developer must pay $\rho\pi$ in the form of either an ex ante license or a court-imposed royalty ex post. Since the incremental value created by B is larger than the expected royalty set by the court, the developer "ignores" the possibility of infringement and adopts B when the patentee does not approach ex ante. Given this, the patentee is indifferent between negotiating a license ex ante or lurking for the developer to adopt and sue for infringement.[15]

For $\rho > \bar{\rho}$, payoffs are a constant function of ρ for both firms. In this range, the developer prefers to adopt the safe technology, A, rather than infringe. Since the patentee gets nothing when the developer adopts A, the patentee approaches for sure and the price of the ex ante license is $\beta\bar{\rho}\pi$. A key intuition is that a sufficiently large ρ makes the developer's threat to use A credible, which *improves* his bargaining position. This insight challenges the conventional wisdom that higher royalties always give non-practicing entities stronger incentives to lurk.

With a certain threat, the patentee's equilibrium expected payoff never strictly exceeds $\bar{\rho}\pi$, i.e., the patentee never receives more than the incremental value of B. We say that the patentee is *over-rewarded* when its expected payoff is larger than $\bar{\rho}\pi$. [16] Thus, the patentee

[15] Any perturbation that breaks the patentee's indifference would get rid of equilibria multiplicity.

[16] Our definition of over-reward contrasts with the Lemley and Shapiro (2007) "no holdup" benchmark of $\theta\beta\bar{\rho}\pi$. As in their model, the patentee's equilibrium payoff with $\lambda = 1$ exceeds that benchmark for all cases except for when $\bar{\rho} = 0$ or when the patent is ironclad and $r = \beta$.

never expects to be over-rewarded when the developer knows for sure that there is a patentee.

Uncertain threat. Our main contribution is to study the case $\lambda \in (0,1)$. That is, when not approached, the developer faces *uncertainty* as to whether there is a lurking patentee. As we will show, equilibrium behavior in the dynamic game depicted in Figure 1 is qualitatively different to the benchmark case of $\lambda = 1$.

We use the perfect-Bayesian equilibrium concept (PBE). Formally, a PBE specifies: (1) the developer's belief, $\hat{\lambda}$, that there is a lurking patentee conditional on not being approached by the patentee before adopting a technology; (2) the developer's probability of adopting A, $a^*(\hat{\lambda}) \in [0,1]$, which is optimally chosen conditional on $\hat{\lambda}$; and (3) the patentee's probability $\phi^* \in [0,1]$ of approaching, which is optimally chosen given $a^*(\hat{\lambda})$. Furthermore, when not approached, the developer's belief updates from λ to $\hat{\lambda}$ according Bayes' rule:

$$\hat{\lambda} = \frac{(1-\phi^*)\lambda}{(1-\phi^*)\lambda + 1 - \lambda}. \tag{1}$$

The updated belief reflects both exogenous uncertainty about whether there is a patentee (λ) and strategic uncertainty about whether patentees tend to approach (ϕ^*).

Choosing A gives the developer a payoff of $(1-\bar{\rho})\pi$, regardless of his belief, $\hat{\lambda}$. Choosing B, gives the developer an expected payoff of $(1-\hat{\lambda}\rho)\pi$. The developer's best response conditional on his belief is to adopt A for sure, $a^*(\hat{\lambda}) = 1$, if $\hat{\lambda}\rho > \bar{\rho}$; to adopt B for sure, $a^*(\hat{\lambda}) = 0$, if $\hat{\lambda}\rho < \bar{\rho}$; and to adopt A with any probability $a^* \in [0,1]$ if $\hat{\lambda}\rho = \bar{\rho}$.

The patentee's expected payoff from approaching with probability ϕ to negotiate a license ex ante is $\phi\Pi_P^{NB} + (1-\phi)(1-a^*(\hat{\lambda}))\rho\pi$.

Proposition 1. *There are three non-generic cases depending on ρ:*[17]

1. *If $\rho \leq \bar{\rho}$, then there is a continuum of payoff-equivalent equilibria in which the developer chooses B for sure and the patentee approaches with some probability $\phi^* \in [0,1]$.*
2. *If $\rho \in \left(\bar{\rho}, \frac{\bar{\rho}}{\lambda}\right)$, then there is a unique equilibrium in which the developer chooses B for sure and the patentee lurks for sure.*
3. *If $\rho \geq \frac{\bar{\rho}}{\lambda}$, then there is a unique equilibrium in which the developer chooses B with probability $1 - a^* = \frac{\beta\bar{\rho}}{\rho}$ and the patentee lurks with probability $1 - \phi^* = \frac{\bar{\rho}(1-\lambda)}{\lambda(\rho-\bar{\rho})}$.*

[17] For the case $\rho = \frac{\bar{\rho}}{\lambda}$, there are two equilibria. One is the limit case of the equilibria described in Proposition 1 part 2. The other one is similar to the part 3, except that project A is chosen with a probability greater than or equal to $\frac{\beta\bar{\rho}}{\rho}$.

Figure 3: Regions with qualitatively different equilibrium behavior

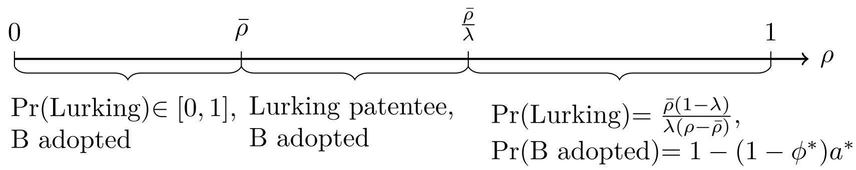

Figure 3 summarizes the three different type of equilibria characterized in Proposition 1. For all values of the expected royalty, patentee lurking and risk-taking by the developer can occur in equilibrium. The central case is for *intermediate* values of the expected royalty, $\rho \in \left(\bar{\rho}, \frac{\bar{\rho}}{\lambda}\right)$, where a developer that is certain that patentees never approach ex ante nonetheless finds the infringement risk of adopting B to be acceptable and a patentee expects to be over-rewarded in ex post licensing. Here, lurking is certain to occur in a unique equilibrium. We start with the intuition for this case.

Consider first the patentee's incentive to lurk. If the developer is certain to adopt B absent an approach, the patentee is clearly better off lurking. An approach would enable a bargain over an ex ante license, which regardless of β caps the patentee's payoff at $\bar{\rho}\pi$. By lurking, the patentee is able to *hold up* the developer, i.e., earn a higher license fee than the developer would have willingly paid ex ante ($\rho\pi > \bar{\rho}\pi$).

The developer faces two scenarios. First, if there is a lurking patentee, a risk-taking developer gets held up in ex post licensing, having already made the technology adoption choice. Had the negotiation taken place ex ante, the developer would have been able to credibly threaten to adopt A. When bargaining for a license ex post, however, the developer's only outside option is to litigate. Therefore, the developer would expect to pay $\rho\pi$ from licensing technology B. The second scenario is that there is no lurking patentee, in which case the developer earns π from adopting B. Even when anticipating that a lurking patentee will be over-rewarded, the developer still chooses B. The reason is that the developer is *uncertain* about whether there is a patentee lurking; if $\rho < \frac{\bar{\rho}}{\lambda}$, it finds the infringement risk from adopting technology B acceptable.

Consider next a *high* expected royalty, $\rho > \frac{\bar{\rho}}{\lambda}$. In this region, if the developer is certain that patentees never approach ex ante, then it finds the infringement risk from adopting technology B unacceptable. Here, it turns out that patentee lurking and risky technology adoption by the developer occur in equilibrium, but less frequently. Intuitively, a high expected royalty yields a classic "cat-and-mouse" game with no pure strategy equilibrium. Start with the developer's incentive to adopt B absent an approach. He anticipates that, if there is a lurking patentee, a license will be too expensive. On the other hand, the developer leaves money on the table by adopting A when there is no patentee. Adopting either A or B for sure when not approached, however, is not an equilibrium. If the developer adopts A for sure, the patentee would get zero from lurking and would get a strictly positive payoff from approaching, so the patentee would approach for sure. Hence, if not approached, the developer would be sure that there is no patentee, so it would prefer to choose B instead of A. Thus, choosing A for sure is not an equilibrium.

Similarly, if the developer's strategy is to adopt B for sure, the patentee would get a higher payoff by lurking and holding up the developer than by approaching, so the patentee lurks for sure. Hence, if not approached, the developer would believe that with probability λ there is a lurking patentee, so it would find the infringement risk unacceptable and choose A instead of B, since $(1-\lambda\rho)\pi < (1-\bar{\rho})\pi$. Thus, choosing B for sure is also not an equilibrium.

Likewise in equilibrium, the patentee cannot approach or lurk for sure. Lurking trades off excessive rents from holdup (when lurking) versus the risk of getting zero when the developer chooses A. If the patentee approaches for sure, the developer would choose B for sure when not approached, so the patentee would have incentives to deviate and lurk instead of approaching. If the patentee lurks for sure, the developer would choose A for sure when not approached, in which case the patentee would rather approach. In the unique equilibrium in mixed strategies, the patentee approaches with a probability such that the developer is indifferent between A and B.

Consider finally a *low* expected royalty, $\rho \leq \bar{\rho}$. In this region, if the developer is certain that there is a patentee, then even without a license it prefers to adopt B. Here, patentee lurking and risky technology adoption choices by the developer obtain as an equilibrium, but the equilibrium is not unique. Now, approaching costs the patentee nothing, as it results in an outcome that is essentially the same as Lemma 2. If bargaining breaks down, the developer will ignore the possibility of infringement and use technology B. This adoption choice is also made in an efficient bargain. Thus, there is no bargaining surplus, and the patentee's payoff $\rho\pi$ is the same as if it lurks. Notice that when $\rho > \bar{\rho}$, uncertainty *qualitatively* changes the behavior of the patentee and the developer.

An important message from Proposition 1 is that technology adoption can be inefficient only if the developer is uncertain and the expected royalty ρ is high. If $\rho \leq \frac{\bar{\rho}}{\lambda}$, the developer always adopts B in equilibrium. If $\rho > \frac{\bar{\rho}}{\lambda}$, technology adoption is inefficient with probability $a^*(1-\phi^*) > 0$.[18] In this inefficient outcome, the developer sometimes chooses A and the

[18] The third case in Proposition 1 is feasible only when $\lambda \in (\bar{\rho}, 1)$, since $\lambda \leq \bar{\rho}$ implies $1 \leq \frac{\bar{\rho}}{\lambda}$. Thus, mixed

patentee sometimes lurks. Incomplete information is a necessary condition for this possibility; if $\lambda = 1$, then technology adoption is always efficient. Furthermore, the probability of inefficient adoption increases when the patentee's bargaining power decreases. A decrease in the prior probability that there is a patentee or in the incremental value of B relative to A, can increase or decrease (to zero) the probability of inefficiency, depending on the value of ρ.

Figure 4 plots the developer's equilibrium probability of adopting B for the case where $\lambda < 1$.. This probability is non-monotonic as a function of ρ. It equals 1 when $\rho < \bar{\rho}/\lambda$, i.e., the region where the equilibrium is in pure strategies. When $\rho < \bar{\rho}/\lambda$, i.e., the region where the equilibrium is in mixed strategies, the developer's probability of adopting B is $1-(1-\phi^*)a^* \in (0,1)$, which is increasing in ρ. This is because an increasing ρ increases the probability of ex ante licensing (by increasing ϕ^*) by more than the decrease in the probability of adopting B.

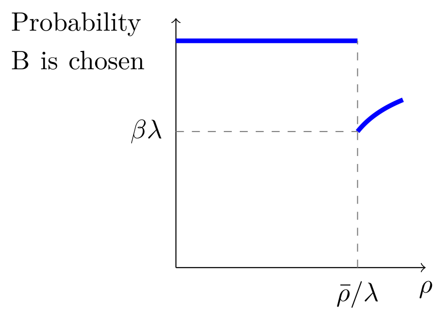

Figure 4: Probability that technology B is chosen in equilibrium.

The proposition below identifies the firms' equilibrium payoffs when $\lambda \in (0,1)$.

Proposition 2. *The developer's ex-ante equilibrium expected payoff is*

$$\Pi_D^* = \begin{cases} (1-\lambda\rho)\pi & \text{if } \rho < \frac{\bar{\rho}}{\lambda}, \\ (1-\bar{\rho})\pi + \left(\frac{\lambda\rho-\bar{\rho}}{\rho-\bar{\rho}}\right)(1-\beta)\bar{\rho}\pi & \text{if } \rho \geq \frac{\bar{\rho}}{\lambda}. \end{cases} \tag{2}$$

The patentee's interim equilibrium payoff is

$$\Pi_P^* = \begin{cases} \rho\pi & \text{if } \rho < \frac{\bar{\rho}}{\lambda}, \\ \beta\bar{\rho}\pi & \text{if } \rho \geq \frac{\bar{\rho}}{\lambda}, \end{cases} \tag{3}$$

When the patentee has all the bargaining power, i.e., $\beta = 1$, the payoffs are identical to the

strategies arise only when the ex-ante probability there is a patentee is larger than the fraction of profits lost from using A instead of B.

benchmark case with a royalty rate of $\hat{r} = \lambda r$ instead of r. In other words, the expected payoffs under uncertainty are the same expected payoffs of a setting with certainty and with *lower* court-imposed royalties. While the payoff are the same, equilibrium behavior is completely different, e.g., inefficient technology adoption occurs only when the developer is uncertain about the presence of the patentee.

When $\beta < 1$, however, the developer's payoff increases with ρ when the expected royalty is high (i.e., when the players are mixing), which is shown in Figure 5 (right panel). In this region, an increase in ρ induces the developer to choose A more frequently, which forces the patentee to increase its probability of approaching.

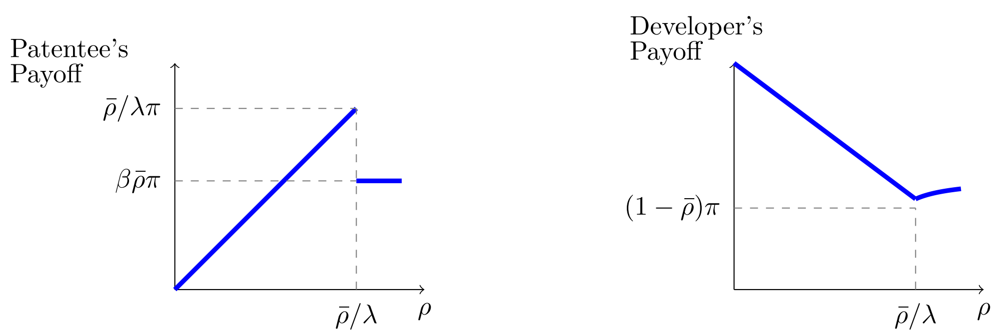

Figure 5: Left: Patentee's interim equilibrium payoff. Right: Developer's ex ante expected equilibrium payoff. The figures consider a moderately likely infringement threat, $\lambda \in (\bar{\rho}, 1)$.

In Figure 5 (left panel), note that the patentee is over-rewarded for any $\rho \in (\bar{\rho}, \bar{\rho}/\lambda)$: it extracts a payoff in excess of the value conferred to the developer ($\rho\pi > \bar{\rho}\pi$). By contrast, in the mixed strategy case ($\rho \geq \frac{\bar{\rho}}{\lambda}$), the patentee is not over-rewarded, as its expected payoff is just $\beta\bar{\rho}\pi$. However, if we condition on delayed enforcement (which occurs with probability $1-\phi^*$), then the resulting payoff for the patentee is $\rho\pi$, which *does* over-reward the patentee. Thus, in the mixed equilibrium case, the patentee's reward at the interim stage is the same as its payoff from ex-ante bargaining (i.e., the patentee's payoff is "reasonable"). However, the patentee's payoff conditional on delayed enforcement is excessive.

Our results uncover an important selection problem when empirically analyzing data on infringement lawsuits: When the expected royalty is high, lawsuit data will not account for cases where the developer could have selected an infringing technology but did not. In those unobserved cases, lurking patentees get zero, while in the observed cases patentees are over rewarded. Also, in practice, it is difficult to observe data on ex ante licensing. Also, Figure 3 shows that there will be *more* lurking when the incremental value of technology B is *smaller*, i.e., when $\bar{\rho}$ is smaller. The reason is that if B was much more valuable than A, then the developer would simply "rationally ignore" the possibility of infringement, since there is too much to miss out by not adopting B.

2.1 Technology Investment and Adoption

We have so far assumed that the developer has already discovered technology B when the game begins. We now consider an earlier investment stage in which the developer has not yet discovered it, but can invest in R&D for the chance to do so. If the developer's efforts fail, then it will adopt A for sure and the game ends. If the developer succeeds in discovering B, then the continuation game is the baseline game from the previous section. We focus on how strategic behavior in that baseline game affects the developer's incentives for R&D investment.

To discover technology B with probability α, the developer must invest $c(\alpha)$, where $c(0) = 0$, and $c(\cdot)$ is strictly increasing and strictly convex. If he fails to develop B, he will adopt technology A. Thus, the developer's optimal ex ante investment maximizes

$$(1-\bar{\rho})\pi + \alpha(\Pi_D^* - (1-\bar{\rho})\pi) - c(\alpha). \tag{4}$$

Thus, the developer's incentive to invest in B with lurking patentees is $\Pi_D^* - (1-\bar{\rho})\pi$, i.e., the difference between his ex ante equilibrium payoff in the game conditional on a patentee that strategically delays enforcement, Π_D^*, and his payoff from adopting technology A for sure, $(1-\bar{\rho})\pi$. Proposition 2, equation (2), shows the expression for Π_D^*, which is independent of the equilibrium probability of approach, ϕ^*, when $\rho \leq \bar{\rho}/\lambda$, and is strictly increasing in ϕ^* when $\rho \geq \bar{\rho}/\lambda$. Using these expressions, a solution to (4) (assumed to be interior) is

$$\alpha^* = \begin{cases} g\left(\bar{\rho} - \lambda\rho\right) & \text{if } \rho < \frac{\bar{\rho}}{\lambda}, \\ g\left(\lambda\phi^*(1-\beta)\bar{\rho}\pi\right) & \text{if } \rho \geq \frac{\bar{\rho}}{\lambda}, \end{cases} \tag{5}$$

where $g(x) \equiv [c']^{-1}(x)$. Note that $g(\cdot)$ is strictly increasing, due to the convexity of $c(\cdot)$.

We say that lurking leads to inefficiency in technology investment if α^* is lower than it would be if the patentee approached for sure ($\phi = 1$). Conditional on B being discovered, we say that lurking leads to inefficiency in technology adoption if the probability that the developer adopts technology B is lower than it would be if the patentee approached for sure. The proposition below describes when these inefficiencies arise.

Proposition 3. *In the game with an investment stage, lurking leads to inefficiency in the following circumstances:*

1. *If $\rho \leq \bar{\rho}$, then lurking does not lead to inefficiency in either investment or adoption.*
2. *If $\rho \in (\bar{\rho}, \frac{\bar{\rho}}{\lambda})$, then lurking leads to inefficiency in investment, but not in adoption.*
3. *If $\rho \geq \frac{\bar{\rho}}{\lambda}$, then lurking leads to inefficiency in both investment and adoption.*

Relative to existing literature, this provides a more detailed and nuanced picture of how delayed enforcement affects efficiency. First, although it does often lead to inefficient investment, this effect is not categorical, in contrast to what the literature usually presumes. With a low expected royalty, investments are efficient even when the developer expects lurking. Additionally, delayed enforcement leads to inefficiency in adoption only if the expected royalty is high. It makes sense that adoption may be efficient even when investment is not (i.e., with an intermediate expected royalty), since the adoption decision is made when the relevant investment has already been sunk.

Note that some possible inefficiencies are independent of the strategic behavior of the patentee. First, there are duplication costs, since knowing the patent would save the investment to re-discover B. Second, with some probability the developer fails to develop B, so he adopts A, which is an inferior technology.[19]

3 Extensions

3.1 The Developer's Search Incentives

In the baseline model, the developer cannot take the initiative to find the patentee and negotiate a license ex ante. In practice, developers can pay for a "freedom to operate" (FTO) search before introducing a new technology. This helps to determine if there are any patents that are likely to be asserted against them in the future. A FTO search removes uncertainty by identifying the patentee which triggers ex ante licensing. In this section, the developer may choose to do a FTO search at the beginning of the game. If the developer

[19] Online Appendix A discusses an alternative timing, where the patentee first decides to approach or lurk, and the developer can to invest to develop B only when the patentee does not approach. Thus, in this section, we focus on the inefficiencies caused solely by the patentee's strategic lurking.

searches and finds a patentee, then the game proceeds as if $\lambda = 1$. If the developer decides not to search, the game in Section 2 is then the subgame.

The developer can conduct a FTO search at some cost $K_D > 0$. The FTO search is assumed to be "perfect," in the sense that the developer will learn about any patent that could be enforced against it. Therefore, the search will identify a patent threat with probability λ, in which case the parties negotiate and the developer gets Π_D^{NB}. With probability $1 - \lambda$, the developer knows for sure that technology B is as safe as A, and therefore its payoff is π.

Thus, we have the following result:

Proposition 4. *There exists ρ_S^L and ρ_S^R, with $\bar{\rho} \leq \rho_S^L < \rho_S^R \leq 1$, such that a developer performs a "Freedom to Operate" search if and only if $K_D \leq (1 - \lambda\beta)\bar{\rho}\pi$ and $\rho \in (\rho_S^L, \rho_S^R)$.*

Figure 6 shows the value of searching for the developer, showing that search incentives are non-monotonic. The maximum value of searching is $(1-\lambda\beta)\bar{\rho}\pi$, and it is attained at $\rho = \bar{\rho}/\lambda$. When the expected royalty ρ is low, a FTO search provides no value. This is because the only benefit of searching is to avoid excessive fees ($\rho > \bar{\rho}$) due to holdup. An implication is that when patent quality θ is sufficiently low, the developer has no incentive to search. These results are largely consistent with Lemley (2008), who notes that in many industries innovative firms generally ignore patents held by other firms. Specifically, technology companies generally instruct engineers not to read patents, adopt policies of ignoring initial demand letters, and choose not to substitute non-infringing redesigns (that are ready to go) until litigation is lost (Lemley, 2008, p.21-22).[20] Chien (2014) also discusses the role of developers ignoring patents, which the author calls "holding out" strategies.

Second, the value of search is non-monotonic for $\rho > \bar{\rho}$. The reason is that for a high expected royalty ($\rho > \bar{\rho}/\lambda$) the patentee approaches the developer with some probability, in contrast to the intermediate expected royalty case ($\rho \in (\bar{\rho}, \bar{\rho}/\lambda)$) when the patentee always lurks. Thus, the value of searching decreases.

[20] Lemley (2008) notes that Intel, Cisco and Microsoft were routinely facing a collective 100 infringement suits at a time.

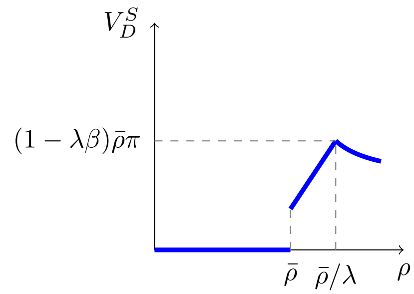

Figure 6: Developer's value of conducting a 'Freedom to Operate' search.

We can use the developer's value of search to devise a mechanism that palliates the lurking problem. Suppose that the developer can offer a monetary *prize* to the patentee on top of the Nash bargaining license if the patentee approaches ex ante. The maximum prize the the developer is willing to pay for any fixed ρ is precisely the value of search depicted in Figure 6.

3.2 The Patentee's Search Incentives

In our baseline model, the patentee is informed about the developer's intentions before the developer makes a technology choice. In practice, patentees can be informed ex-post about a developer infringing on its patent. To accommodate for this case, let there be two types of patentees: one that is informed early (type E) and one that is informed late (type L). Suppose that it is common knowledge that the frequency of these types are λ_E and λ_L, respectively, with $\lambda_E + \lambda_L = \lambda$. The difference with the baseline model is type L. This patentee type discovers infringement *after* the technology choice was made, and therefore its only option to filing a lawsuit ex post. Thus, the patentee of type L's payoff is simply $\rho\pi$, and the developer gets $(1-\rho)\pi$ when choosing B and facing a patentee of type L.

The developer's information set when not approached by the patentee before choosing a technology has now three nodes: one for each patentee type and one for no patent threat. The developer forms beliefs over these nodes, which we call $\hat{\lambda}_E$, $\hat{\lambda}_L$, and $\hat{\lambda}_N$. If the strategy of the patentee informed earlier is to approach the developer with probability ϕ, then Bayes' rule implies that the developer beliefs are

$$\hat{\lambda}_E = \frac{\lambda_E(1-\phi)}{\lambda_E(1-\phi) + \lambda_L + \lambda_N}.$$

A patentee informed late is non strategic and never approaches early, so

$$\hat{\lambda}_L = \frac{\lambda_L}{\lambda_E(1-\phi) + \lambda_L + \lambda_N},$$

and $\hat{\lambda}_N = 1 - \hat{\lambda}_E - \hat{\lambda}_L$. The developer's payoff from choosing A is the same as in the baseline case. The developer's payoff from choosing B, however, is

$$(\hat{\lambda}_E + \hat{\lambda}_L)(1-\rho)\pi + (1 - (\hat{\lambda}_E + \hat{\lambda}_L))\pi = (1 - (\hat{\lambda}_E + \hat{\lambda}_L)\rho)\pi.$$

The developer knows that she will be liable for damages after choosing B when she faces a patentee of type E or L. The informed patentee's expected payoff from approaching with probability ϕ to negotiate a license ex ante is $\phi\Pi_P^{NB} + (1-\phi)(1 - a^*(\hat{\lambda}_E, \hat{\lambda}_L))\rho\pi$.

Proposition 5. *There are four non-generic cases depending on ρ:*

1. *If $\rho \leq \bar{\rho}$, then there are multiple, payoff-equivalent equilibria where the developer chooses B for sure and the patentee approaches with probability $\phi^* \in [0, 1]$.*
2. *If $\rho \in \left(\bar{\rho}, \frac{\bar{\rho}}{\lambda_E + \lambda_L}\right)$, then there is a unique equilibrium in which the developer chooses B for sure and the patentee lurks for sure.*
3. *If $\rho \in \left(\frac{\bar{\rho}}{\lambda_E+\lambda_L}, \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}\right)$, then there is a unique equilibrium in which the developer chooses B with probability $a^* = \frac{\beta\bar{\rho}}{\rho}$ and the patentee lurks with probability $1 - \phi^* = \frac{\bar{\rho}(1-\lambda_E) - \rho\lambda_L}{\lambda_E(\rho - \bar{\rho})}$.*
4. *If $\rho \geq \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}$, then there is an inefficient equilibrium in which the developer chooses A for sure and the patentee approaches to license ex ante whenever informed.*

Note that when $\lambda_L \to 0$, Proposition 5 collapses to Proposition 1. When $\lambda_L > 0$, Proposition 5 (part 4) shows that it is possible that the developer adopts A *even if* the patentee of type E has no intention to strategically lurk. The reason is that the patentee of type L gets over compensated (they receive more than $\bar{\rho}$), but it does not find the developer in time to prevent her from choosing the inefficient technology.

Proposition 6. *When $\rho < \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}$, the patentee is indifferent among any combination (λ_E, λ_L) such that $\lambda_E + \lambda_L = \lambda$, while for $\rho \geq \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}$ the patentee's strictly prefers $(\lambda, 0)$ over any other combination.*

When $\rho \leq \frac{\bar{\rho}}{\lambda_E+\lambda_L}$, the developer chooses B regardless of whether the patentee approaches early or late. Furthermore, the patentee does not benefit from finding out about the developer since lurking and arriving late is the optimal strategy. When $\rho \in \left(\frac{\bar{\rho}}{\lambda_E+\lambda_L}, \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}\right)$ the patentee mixes between lurking and approaching. However, the mixing probability captures the possibility of late arrival, $\phi^* = \frac{\rho}{\lambda_E(\rho-\bar{\rho})}$, i.e., the patentee approaches more often when it is more likely to arrive late. In equilibrium, the patentee's payoff is independent of the combination (λ_E, λ_L) when there is mixing. Lastly, when $\rho \geq \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}$, the patentee's best scenario is to find potential infringers as soon as possible (i.e., $\lambda_E = \lambda$, as in the baseline case) because otherwise it gets 0 since the developer chooses A.

Corollary 1. *The patentee is willing to spend resources searching suitable developers early on only when* $\rho \geq \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}$.

When $\rho \geq \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}$, if the patentee does not search, the developer is always going to adopt A, while searching can lead to B being adopted. The expected value of searching is constant and equal to $(1-\beta\lambda\bar{\rho})\pi - (1-\bar{\rho})\pi = (1-\beta\lambda)\pi\bar{\rho}$. The patentee will search for high values of ρ because lurking leads to the adoption of A, since the developer faces unacceptable risk in choosing B. Thus, it is in the patentee's best interest to identify suitable targets as soon as possible. Of course, as we show in the baseline model, once the patentee has that information, it will use it strategically, i.e., it may lurk.

3.3 The Developer's Defense: Declaratory Judgment

In the baseline game, if the patentee approaches the developer ex ante and negotiations break down, the developer can either adopt technology B "at risk" or pick the safe technology. In practice, however, the developer could instead take the initiative and file declaratory judgment action—a preemptive suit to assess whether the patent would be infringed. This allows the developer "who is reasonably at legal risk because of an unresolved dispute, to obtain judicial resolution of that dispute without having to await the commencement of legal action by the other side."[21]

To analyze the impact of declaratory judgment on the patentee's delay incentives, we expand the baseline model by allowing a developer that is approached before choosing a technology to file for declaratory judgment (DJ). We assume that DJs are costless and the probability distribution over outcomes is the same as in an infringement suit: with probability θ the

[21]Elecs. for Imaging, Inc. v. Coyle, 394 F.3d 1341, 1345 (Fed. Cir. 2005) (quoting BP Chems. Ltd. v. Union Carbide Corp., 4 F.3d 975, 977 (Fed.Cir.1993)).

patentee wins and the developer would face a liability of royalty r if it uses B without a license, and with probability $1-\theta$ the patentee loses and the developer can use B without liability.

The developer's payoff from not exercising the DJ option is the same as in the baseline. If it exercises the DJ option, all infringement uncertainty is resolved *before* a technology choice is made. If the outcome of the DJ rules against the patentee, ruling that the developer is free to use B, then the developer will use B and get a payoff of π. Otherwise, if the outcome of the DJ rules in favor of the patentee, ruling that the developer will certainly infringe, then the developer chooses between A and B, except that now the payoff from using B is $(1-r)\pi$ instead of $(1-\rho)\pi$ (recall that $\rho = \theta r$).

Following an unfavorable ruling for the developer after the DJ, the parties will engage in a Nash bargaining negotiation, where if $r > \bar{\rho}$, the developer can credibly threaten to choose A. Thus, the Nash bargaining payoffs are

Lemma 3. *If the declaratory judgment rules that technology* B *infringes, Nash bargaining results in the following payoffs:*

$$\Pi_P^{NB,DJ} = \begin{cases} r\pi & \text{if } \rho \leq \theta\bar{\rho} \\ \beta\bar{\rho}\pi & \text{if } \rho > \theta\bar{\rho} \end{cases}, \quad \Pi_D^{NB,DJ} = \begin{cases} (1-r)\pi & \text{if } \rho \leq \theta\bar{\rho}, \\ (1-\beta\bar{\rho})\pi & \text{if } \rho > \theta\bar{\rho}. \end{cases}$$

If the DJ results in an unfavorable outcome for the developer, the negotiation payoffs are equivalent to those in Lemma 1 with a worse alternative ($\theta\bar{\rho}$ instead of $\bar{\rho}$) and lower payoffs for the developer in the region $\rho \leq \theta\bar{\rho}$.

Proposition 7. *The developer has strict incentives to file a DJ whenever $\rho > \theta\bar{\rho}$, and is indifferent between filing a DJ and not, otherwise. The the patentee's and the developer's expected payoff, respectively, are*

$$\Pi_P^{DJ} = \begin{cases} \rho\pi & \text{if } \rho \leq \theta\bar{\rho} \\ \theta\beta\bar{\rho}\pi & \text{if } \rho > \theta\bar{\rho}, \end{cases}, \quad \Pi_D^{DJ} = \begin{cases} (1-\rho)\pi & \text{if } \rho \leq \theta\bar{\rho}, \\ (1-\theta\beta\bar{\rho})\pi & \text{if } \rho > \theta\bar{\rho}. \end{cases}$$

Comparing these payoffs with those in Lemma 1 it is clear that the developer benefits precisely when the expected royalty is low ($\rho < \bar{\rho}$) *only* because of uncertainty about the patent strength ($r > \bar{\rho}$). In such a case, a developer approached by a patentee would file a DJ to resolve this uncertainty. The patentee then has less incentive to approach and more incentive to lurk.

Allowing the developer to file a DJ is equivalent to a modified version of the baseline where product A's payoff is $(1-\theta\bar{\rho})\pi$ rather than $(1-\bar{\rho})\pi$. Therefore, we can easily analyze the impact of DJs on all the extensions of the baseline model.

3.4 Costly Litigation

In the baseline model, litigation is costless. Here, we explore how costly litigation affects lurking in equilibrium. We assume the litigation cost of the developer and the patentee are, respectively, $c_D\pi$ and $c_P\pi$.

The patentee's payoff from litigation is $(\rho - c_P)\pi$, so litigation is credible only when $\rho > c_P$. If litigation is not credible, then the developer always chooses B for sure. When litigation is credible, the parties will settle. Saving on litigation costs creates an additional surplus for parties to bargain over. The patentee extracts a fraction β of that surplus. Let us define the patentee's "litigation-cost advantage" as $\gamma \equiv \beta(c_P + c_D) - c_P$. This term reflects that litigation costs change the patentee's settlement payoff by $\gamma\pi$. When $\gamma > 0$, the patentee's settlement is larger relative to costless litigation. For instance, when $c_P = 0$ and $c_D > 0$, $\gamma = \beta c_D > 0$. With equal litigation costs and equal bargaining ability $\gamma = 0$. Using these payoffs, we can write down a version of Proposition 1 with costly litigation. To narrow down the number of cases to be analyze, let us assume that $c_P < \bar{\rho} - \gamma$.

Proposition 8. *There are three non-generic cases depending on ρ:*

1. *If $\rho \leq \bar{\rho} - \gamma$, then there is a continuum of payoff-equivalent equilibria in which the developer chooses B for sure and the patentee approaches with some probability $\phi^* \in [0,1]$ when $\rho \in [c_P, \bar{\rho} - \gamma]$ and never approaches when $\rho < c_P$.*

2. *If $\rho \in \left(\bar{\rho} - \gamma, \frac{\bar{\rho}}{\lambda} - \gamma\right)$, then there is a unique equilibrium in which the developer chooses B for sure and the patentee lurks for sure.*

3. *If $\rho \geq \frac{\bar{\rho}}{\lambda} - \gamma$, then there is a unique equilibrium in which the developer chooses B with probability $1 - a^* = \frac{\beta\bar{\rho}}{\rho+\gamma}$ and the patentee lurks with probability $1 - \phi^* = \frac{\bar{\rho}(1-\lambda)}{\lambda(\rho+\gamma-\bar{\rho})}$.*

When the patentee has a litigation-cost advantage (i.e., $\gamma > 0$) the region where lurking is strictly profitable shifts to the left. In other words, lurking is now strictly optimal for some low levels of the expected royalty ($\rho < \bar{\rho}$) where absent litigation costs the patentee does not benefit strictly from lurking. Also, there is a larger region where the equilibrium is in mixed strategies, i.e., the patentee's litigation-cost advantage can lead to more inefficient adoption. The opposite happens when $\gamma < 0$. This is the case, for instance, when β is low. In that case, the region where lurking is strictly profitable shifts to the right, reducing the region where mixing leads to inefficient adoption.

Thus, our model can rationalize that a patentee is more willing to lurk when it has a litigation-cost advantage. In fact, with a sufficient litigation-cost advantage (specifically, $\bar{\rho} < \gamma$) a patentee with a credible litigation always lurks with positive probability.

3.5 Developer's Strategic Commitment

When $\rho > \bar{\rho}/\lambda$, adoption is inefficient. If the developer can make a credible *commitment* to adopt A if not approached, then the patentee would always approach. Adopting A for sure, however, is not a perfect-Bayesian equilibrium. Does the developer benefit from commitment? The payoff from committing to adopt A if not approached is $\lambda(1-\beta\bar{\rho})\pi + (1-\lambda)(1-\bar{\rho})\pi$, whereas the developer's expected payoff in the perfect-Bayesian equilibrium is $(1-\bar{\rho})\pi + \lambda\phi^*(1-\beta)\bar{\rho}\pi$. Comparing these two expressions, we find that commitment is strictly better for the developer whenever $\beta < 1$.

Therefore, if possible, the developer would credibly commit to adopt A if not approached, in which case the patentee would never lurk. Commitment avoids lurking but does not restore efficiency, since the developer will adopt A for sure when there is no patentee. When $\rho > \bar{\rho}/\lambda$, there is more nuance. Commitment achieves efficient adoption with probability λ, whereas a PBE achieves efficiency with probability $1 - a^*(1-\phi^*)$. Thus, commitment enhances efficiency whenever

$$1 - \lambda < a^*(1-\phi^*) \iff \lambda\rho(\rho - \bar{\rho}) < (\rho - \beta\bar{\rho})\bar{\rho}.$$

This inequality holds when ρ is close to $\bar{\rho}/\lambda$, so it holds whenever $\rho \in (\bar{\rho}/\lambda, \hat{\rho}]$, where $\hat{\rho}$ depends on $\bar{\rho}$ and λ.[22]

[22]Note that when $\rho \in \left(\bar{\rho}, \frac{\bar{\rho}}{\lambda}\right)$, commitment clearly reduces efficiency relative to the PBE. The developer always picks B in the PBE, but picks B with probability λ under commitment.

4 Discussion

The literature on patent holdup explains that a patent owner can typically extract excessive royalties if it waits to enforce a patent until after an unwitting infringer has adopted the patented technology. It contrasts this with the case of ex ante licensing, where no such problem arises. However, merely comparing these two scenarios does not tell us what will actually occur in equilibrium. Rational product developers will respond strategically to the risk of patent holdup by potentially modifying their decisions about technology investment or adoption. And this, in turn, will shape a patentee's incentives about whether to "lurk" (i.e. to attempt holdup) in the first place. The main contribution of our paper is to endogenize these behaviors, allowing us to specify precisely when holdup occurs in equilibrium, and how the risk of holdup affects equilibrium technology choices.

Our results show that the prevalence and effects of strategic lurking are more subtle than conventionally assumed in existing literature. While the literature tends to suggest that patentees almost always want to lurk (due to the higher royalties associated with holdup), we show that incentives to lurk are more limited and nuanced. There is a natural limit on how widespread strategic lurking can be, for if the risk of holdup gets sufficiently large developers will stop adopting risky technologies, and this eliminates the strategic value of lurking. Similarly, the literature tends to assume that the risk of holdup will always have an adverse effect on technology choices. We show that, while it is true that lurking often reduces incentives for R&D investment, this is not always true; and lurking frequently has no adverse effect on technology adoption choices.

At the same time, our findings do not support one popular critique of the holdup literature, which is that holdup will not occur if product developers are rational and can anticipate the risk ex ante.[23] In other words, this critique asserts that, if developers' technology choices were left endogenous, then holdup would not arise in equilibrium. Our results show this is not the case, although the emergence of holdup in equilibrium is more limited than the literature tends to suggest.

As a policy matter, our results suggest that holdup is indeed problematic, and that certain legal interventions could be helpful to address it. We discussed the use of declaratory judgment actions as one potentially helpful option. An even more obvious possibility would be to reduce or eliminate a patentee's damages award if a court finds that it unreasonably delayed

[23]E.g., Galetovic and Haber (2017) assert that "in patent holdup theory...firms cannot adapt and solve the problem wrought by [holdup] because the game begins after...manufacturers invest. Adaptations to prevent holdup are ruled out by construction."

enforcement. In fact, such a rule used to exist.[24] However, the Supreme Court struck down that rule in 2017.[25] Our findings suggest that new legislation reviving the doctrine would be helpful.

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[24] It stemmed from the legal doctrine of "laches," but the Supreme Court held that it does not apply to the Patent Act.

[25] SCA Hygiene Prods. Aktiebolag v. First Quality Baby Prods., LLC, 137 S.Ct. 954 (2017).

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A Proofs

Proof of Lemma 1

Proof. When the patentee approaches before the developer has chosen a technology, the developer threaten to use technology A, which is credible only when the developer's expected payoff from using A is larger than her payoff from using B. If indifferent, we assume that the developer always chooses A. Using technology A is a credible threat if and only if

$$(1-\bar{\rho})\pi \geq (1-\rho)\pi \Leftrightarrow \rho \geq \bar{\rho}.$$

When $\rho \geq \bar{\rho}$, an agreement increases the joint surplus by $\pi - (1-\bar{\rho})\pi = \bar{\rho}\pi$ because the developer uses the efficient technology, B. Under Nash bargaining, the patentee and the developer splits this surplus, with the patentee getting a share β. When $\rho > \bar{\rho}$ the developer does not have a credible threat, chooses technology B, and gets $\rho\pi$. □

Proof of Lemma 2

Proof. The developer chooses B if $\rho < \bar{\rho}$, chooses A if $\rho > \bar{\rho}$ and is indifferent if $\rho = \bar{\rho}$. The patentee is indifferent about approaching and lurking when $\rho < \bar{\rho}$, as there is no bargaining surplus. Therefore, for $\rho < \bar{\rho}$ there are multiple equilibria: the developer chooses B for sure and the patentee approaches with probability $\phi^* \in [0,1]$. When $\rho > \bar{\rho}$, the patent holder approaches for sure, as otherwise it would earn a payoff of zero. When $\rho = \bar{\rho}$, the developer mixes between A and B, choosing A with probability a^*. Approaching with probability ϕ gives the patentee $(1-\phi)(1-a^*)\rho\pi + \phi\rho\pi$, so choosing $\phi^* = 1$ is optimal when $a^* > 0$. □

Proof of Proposition 1

Proof. We have three cases:

(a) Separating equilibrium, $\phi^* = 1$. When a patentee always approach, we have $\hat{\lambda} = 0$, and therefore the developer chooses B. The patent holder's payoff from deviating and choosing to approach with a different probability ϕ is $\rho\pi + \phi(\Pi_P^{NB} - \rho\pi)$. We can sustain $\phi^* = 1$ in equilibrium only if $\Pi_P^{NB} \geq \rho\pi$. When $\rho \leq \bar{\rho}$, this condition holds with equality. When $\rho > \bar{\rho}$, this condition is $\beta\bar{\rho}\pi \geq \rho\pi$, or $\beta\bar{\rho} \geq \rho$, which cannot hold since it would imply that $\beta\bar{\rho} \geq \rho > \bar{\rho}$.

(b) Pooling equilibrium, $\phi^* = 0$. When the patentee lurks for sure, we have $\hat{\lambda} = \lambda$. Under this belief, the developer chooses A whenever $\lambda\rho > \bar{\rho}$. If this condition holds, the patentee receives zero, so it would prefer to deviate to 'Approach.' Thus, for $\phi^* = 0$ to be an equilibrium, it must be that $\lambda\rho \leq \bar{\rho}$. If $\lambda\rho < \bar{\rho}$, the developer chooses B, and the patentee gets $\rho\pi$ by not approaching. The patentee must no have incentives to deviate, which happens when $\rho\pi \geq \Pi_P^{NB}$. This condition holds with equality for $\rho \leq \bar{\rho}$ and it also holds for $\rho > \bar{\rho}$ because $\rho > \bar{\rho} \Rightarrow \rho \geq \beta\bar{\rho}$. Thus, when $\lambda\rho < \bar{\rho}$, $\phi^* = 0$ is an equilibrium. Lastly, when $\lambda\rho = \bar{\rho}$, the developer is indifferent between A and B, so suppose the developer chooses A with probability $a^* \in [0, 1]$. In that case, the patentee's expected payoff from lurking is $(1-a^*)\rho\pi$, while the payoff from approaching is Π_P^{NB}. For $\rho \leq \bar{\rho}$, the patentee is indifferent when $a^* = 0$. For $\rho > \bar{\rho}$, the patentee prefers to lurk iff $(1 - a^*)\rho\pi \geq \beta\bar{\rho}\pi$ or, equivalently, $a^* \leq 1 - \frac{\beta\bar{\rho}}{\rho}$. Thus, $\phi^* = 0$ is an equilibrium whenever $\lambda\rho \leq \bar{\rho}$.

(c) Semi-pooling, $\phi^* \in (0, 1)$: For the patentee to mix between lurking and approaching, we must have $\Pi_P^{NB} = (1 - a^*(\hat{\lambda}))\rho\pi$. We study two cases:

(c.1) When $\rho > \bar{\rho}$, we need $\beta\bar{\rho} = (1 - a^*(\hat{\lambda}))\rho$. Since $\beta \leq 1$, this can hold only if $a^*(\hat{\lambda}) \in (0, 1)$, meaning that the developer is indifferent between A and B, i.e., when $\hat{\lambda}\rho = \bar{\rho}$. This requires

$$\frac{(1-\phi)\lambda}{(1-\phi)\lambda + 1 - \lambda} = \frac{\bar{\rho}}{\rho} \Leftrightarrow \phi^* = 1 - \frac{\bar{\rho}(1-\lambda)}{\lambda(\rho - \bar{\rho})}.$$

Clearly $\phi^* < 1$. However, $\phi^* > 0$ if and only if $\lambda\rho > \bar{\rho}$.

(c.2) When $\rho < \bar{\rho}$, then $\hat{\lambda}\rho \leq \rho < \bar{\rho}$. Therefore, $a^*(\hat{\lambda}) = 0$ and $\Pi_P^{NB} = \rho\pi$. So in this case, any $\phi^* \in (0, 1)$ will be an equilibrium.

(c.3) When $\rho = \bar{\rho}$, $\Pi_P^{NB} = \rho\pi$. For this to be an equilibrium, the developer must pick B with positive probability and leave the patentee indifferent between approaching or lurking. So we need $\hat{\lambda}\rho \leq \bar{\rho}$, in which case the patent holder gets $(1 - a^*(\hat{\lambda}))\rho\pi$ from lurking. In order for the patentee to not deviate, we must have $a^*(\hat{\lambda}) = 0$, i.e., when indifferent, the developer must choose B. This is always an equilibrium because $\hat{\lambda} \leq 1$.

$\square$

Proof of Proposition 2

Proof. **Equilibrium payoffs.** If tech B is unpatented, the patentee's payoff is zero. Otherwise, the patentee receives a payoff of $\rho\pi$ when $\rho < \frac{\bar{\rho}}{\lambda}$. When $\rho > \frac{\bar{\rho}}{\lambda}$, the patentee is indifferent between lurking and approaching, so its equilibrium payoff is $\beta\bar{\rho}\pi$.

When $\rho < \frac{\bar{\rho}}{\lambda}$, the developer always chooses B and her expected payoff is $\lambda(1-\rho)\pi + (1-\lambda)\pi$. When $\rho > \frac{\bar{\rho}}{\lambda}$ both the patentee and the developer play mixed strategies. The developer's is indifferent between A and B if the patentee does not approach, which happens with probability $1 - \lambda\phi^*$, so her expected equilibrium payoff is $(1 - \lambda\phi^*)(1 - \bar{\rho})\pi + \lambda\phi^*(1 - \beta\bar{\rho})\pi$ which equals $(1 - \bar{\rho})\pi + \lambda\phi^*(1 - \beta)\bar{\rho}\pi$, where $\phi^* = 1 - \frac{\bar{\rho}(1-\lambda)}{\lambda(\rho-\bar{\rho})}$. $\square$

Proof of Proposition 3

Proof. Let us compare the investment incentives and adoption decision in two scenarios. First, consider a game where the developer adopts A for sure if he is unable to develop B. If the developer successfully develops B, the continuation game is the baseline game. The developer's incentive to invest ex ante to discover B is

$$I_{\text{strategic}} = \Pi_D^* - (1 - \bar{\rho})\pi.$$

Alternatively, consider a game where the developer adopts A for sure if he is unable to develop B. If the developer successfully develops B, the patentee *always* approaches (i.e., never lurks), and the parties Nash Bargain. In this game, the developer's incentive to invest ex ante to discover B is

$$I_{\text{always approach}} = \lambda\Pi_D^{NB} + (1 - \lambda)\pi - (1 - \bar{\rho})\pi.$$

Thus, the incentive to invest in both games is the same when $\rho \leq \bar{\rho}$ because, in this case, $\Pi_D^* = \lambda\Pi_D^{NB} + (1 - \lambda)\pi = 1 - \lambda\rho$. Conditional on a successful discovery of B, B will be adopted.

The incentive to invest is larger when the patentee always approaches when $\bar{\rho} \leq \rho \leq \bar{\rho}/\lambda$ because, in this case, $\Pi_D^* = (1 - \lambda\rho)\pi < (1 - \lambda\beta\bar{\rho})\pi = \lambda\Pi_D^{NB} + (1 - \lambda)\pi$, since $\rho > \beta\bar{\rho}$. Conditional on a successful discovery of B, B will be adopted.

The incentive to invest is larger when the patentee always approaches when $\bar{\rho} \leq \rho \leq \bar{\rho}/\lambda$ because, in this case, $\Pi_D^* = (1 - \bar{\rho})\pi + \lambda\phi^*(1 - \beta)\bar{\rho}\pi \leq (1 - \lambda\beta\bar{\rho})\pi = \lambda\Pi_D^{NB} + (1 - \lambda)\pi$, since $\phi^* \leq 1$. Conditional on a successful discovery of B, B will be adopted with probability

$\phi^* < 1$. $\square$

Proof of Proposition 4

Proof. Directly from Lemma 1, the developer's expected payoff from searching is

$$U_D^{FTO} = \begin{cases} (1-\lambda\rho)\pi, & \text{if } \rho \leq \bar{\rho} \\ (1-\beta\lambda\bar{\rho})\pi, & \text{if } \rho > \bar{\rho}. \end{cases} \tag{6}$$

If the developer does not search, the developer expects to receive the equilibrium payoff in Proposition 2, Π_D^*. Thus, the marginal value of searching to the developer is, $V_D^{\text{Search}} = U_D^{FTO} - \Pi_D^*$, where

$$V_D^{\text{Search}} = \begin{cases} 0, & \text{if } \rho \leq \bar{\rho} \\ (\rho - \beta\bar{\rho})\lambda\pi, & \text{if } \bar{\rho} < \rho \leq \bar{\rho}/\lambda \\ [1-\lambda(\phi^* + \beta - \beta\phi^*)]\bar{\rho}\pi, & \text{if } \rho > \bar{\rho}/\lambda. \end{cases} \tag{7}$$

$\square$

Proof of Proposition 5

Proof. We study three cases:

1. Equilibria with $\phi^* = 1$. In contrast to the baseline case, there is residual risk even if patentees always approach ex ante. So, $\hat{\lambda}_E = 0$, $\hat{\lambda}_L = \frac{\lambda_L}{\lambda_L+\lambda_N}$, and $\hat{\lambda}_N = \frac{\lambda_N}{\lambda_L+\lambda_N}$.

   Thus, the developer chooses A whenever $\hat{\lambda}_L\rho > \bar{\rho}$. In the case, the patentee does not have a profitable deviation, so there is inefficiency in equilibrium.

   Thus, the developer chooses B whenever $\hat{\lambda}_L\rho < \bar{\rho}$. In this case, the developer compares $\rho\pi$ with Π_D^{NB}. Whenever $\rho \leq \bar{\rho}$, the developer is indifferent, so this is an equilibrium. However, when $\hat{\lambda}_L\rho < \bar{\rho} < \rho$, the developer always want to deviate because $\beta\bar{\rho}\pi < \rho\pi$.

   Thus, there is an equilibrium in which $\phi^* = 1$ if and only if $\frac{\lambda_L}{\lambda_L+\lambda_N}\rho > \bar{\rho}$, or $\frac{\lambda_L}{\lambda_L+\lambda_U}\rho < \bar{\rho}$ and $\rho \leq \bar{\rho}$.

2. Equilibria with $\phi^* = 0$. Bayes implies $\hat{\lambda}_E = \lambda_E$, $\hat{\lambda}_L = \lambda_L$, and $\hat{\lambda}_N = 1 - \hat{\lambda}_E - \hat{\lambda}_L$. Let $\lambda = \lambda_E + \lambda_L$. From here, the proof is identical to the proof of Proposition 1, so $\phi^* = 0$

and B is chosen by the developer conditional on no approach whenever $\lambda\rho \leq \bar{\rho}$.

3. Semi-pooling, $\phi^* \in (0,1)$: Define $\hat{\lambda} = \hat{\lambda}_E + \hat{\lambda}_L$. For the patentee to mix between lurking and approaching, we must have $\Pi_P^{NB} = (1 - a^*(\hat{\lambda}))\rho\pi$. We study two cases:

   (c.1) When $\rho > \bar{\rho}$, we need $\beta\bar{\rho} = (1 - a^*(\hat{\lambda}))\rho$. Since $\beta \leq 1$, this can hold only if $a^*(\hat{\lambda}) \in (0,1)$, meaning that the developer is indifferent between A and B, i.e., when $\hat{\lambda}\rho = \bar{\rho}$. This requires

   $$\frac{\lambda_E(1-\phi) + \lambda_L}{\lambda_E(1-\phi) + \lambda_L + \lambda_N} = \frac{\bar{\rho}}{\rho} \Leftrightarrow \phi^* = 1 - \frac{\bar{\rho}(1-\lambda_E) - \rho\lambda_L}{\lambda_E(\rho - \bar{\rho})}.$$

   For this probability to be well defined we need $(\lambda_E + \lambda_L)\rho \geq \bar{\rho}$ and in addition we require $\lambda_L\rho \leq \bar{\rho}(1-\lambda_E)$.

   (c.2) When $\rho < \bar{\rho}$, then $\hat{\lambda}\rho \leq \rho < \bar{\rho}$. Therefore, $a^*(\hat{\lambda}) = 0$ and $\Pi_P^{NB} = \rho\pi$. So in this case, any $\phi^* \in (0,1)$ will be an equilibrium.

   (c.3) When $\rho = \bar{\rho}$, $\Pi_P^{NB} = \rho\pi$. For this to be an equilibrium, the developer must pick B with positive probability and leave the patentee indifferent between approaching or lurking. So we need $\hat{\lambda}\rho \leq \bar{\rho}$, in which case the patent holder gets $(1 - a^*(\hat{\lambda}))\rho\pi$ from lurking. In order for the patentee to not deviate, we must have $a^*(\hat{\lambda}) = 0$, i.e., when indifferent, the developer must choose B. This is always an equilibrium because $\hat{\lambda} \leq 1$.

□

Proof of Proposition 6

Proof. The patentee's payoff is independent of the distribution (λ_E, λ_L) whenever $\rho \leq \frac{\bar{\rho}}{\lambda_E + \lambda_L}$. When $\rho \in \left(\frac{\bar{\rho}}{\lambda_E+\lambda_L}, \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}\right)$ the patentee's expected payoff is

$$\Pi_P = [\lambda_L(1-a^*) + \lambda_E(1-a^*)(1-\phi^*)]\rho\beta + \lambda_E\phi^*\Pi_P^{NB}.$$

Using that $\lambda_E = \lambda - \lambda_L$, we get $\Pi_P = \lambda\rho\beta + \lambda_E\phi^*[\Pi_P^{NB} - (1-a^*)\rho\beta]$. Since a^* is independent of λ_L, and from the definition of ϕ^* we have that $\lambda_E\phi^*$ is independent of λ_L we have $\frac{\partial \Pi_P}{\partial \lambda_L} = 0$.

Finally, when $\rho > \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}$ the patentee gets zero. Since for any other value of ρ the patentee gets a strictly positive expected payoff, the patentee is better off whenever λ_L is as small as possible, to shink the region $\rho > \frac{\bar{\rho}(1-\lambda_E)}{\lambda_L}$. □

Proof of Proposition 7

Proof. The developer's expected payoff from filing a DJ is

$$\Pi_D^{DJ} = (1-\theta)\pi + \theta\Pi_D^{NB,DJ} = \begin{cases} (1-\rho)\pi & \text{if } \rho \leq \theta\bar{\rho}, \\ (1-\theta\beta\bar{\rho})\pi & \text{if } \rho > \theta\bar{\rho}. \end{cases}$$

The expected benefit/loss from filing a DJ is therefore given by the difference $\Pi_D^{DJ} - \Pi_D^{NB}$ which is given by

$$\Pi_D^{DJ} - \Pi_D^{NB} = \begin{cases} 0 & \text{if } \rho \leq \theta\bar{\rho}, \\ (\rho - \theta\beta\bar{\rho})\pi & \text{if } \rho \in (\theta\bar{\rho}, \bar{\rho}), \\ -(1-\theta)\beta\bar{\rho}\pi & \text{if } \rho > \bar{\rho}. \end{cases}.$$

It is worth filing a DJ whenever $\Pi_D^{DJ} - \Pi_D^{NB} \geq 0$, which happens only when $\rho > \theta\bar{\rho}$. □

Proof of Proposition 8

Proof. If litigation is credible, then the parties will bargain and the settlement payoffs (the equivalent to Lemma 1 in the main text) are

Lemma 4. *If technology* B *is patented and the patentee approaches ex ante, Nash bargaining results in the following payoffs:*

$$\Pi_P^{NB} = \begin{cases} (\rho+\gamma)\pi & \textit{if } (\rho+\gamma) \leq \bar{\rho} \\ \beta\bar{\rho}\pi & \textit{if } \rho+\gamma > \bar{\rho} \end{cases}, \quad \Pi_D^{NB} = \begin{cases} (1-(\rho+\gamma))\pi & \textit{if } \rho+\gamma \leq \bar{\rho} \\ (1-\beta\bar{\rho})\pi & \textit{if } \rho+\gamma > \bar{\rho} \end{cases}.$$

Using these payoffs, the proof is analogous to the baseline case.

□

Supplemental Material – Intended for Online Publication

Product Development with Lurking Patentees

Erik Hovenkamp, Jorge Lemus and John L. Turner

A Development Cost and Uncertain Success

In this section, we change the timing of the developer's investment decision. If the patentee does not approach, the developer can invest in R&D to develop a working version of tech B. To succeed with probability α, the patentee must invest an amount $c(\alpha)$. If he fails to develop B, he will adopt technology A. When the developer holds a belief $\hat{\lambda}$, conditional on not being approached, he maximizes

$$\Pi_{NA}(\rho, \alpha) = V_A + \alpha(V_B - V_A) - c(\alpha),$$

where $V_A = (1-\bar{\rho})\pi$ and $V_B = (1-\hat{\lambda}\rho)\pi$. Denote by $\alpha^*(\hat{\lambda}, \rho) \in [0, 1]$ the solution to this problem, which is weakly decreasing in $\hat{\lambda}$. Whenever $\bar{\rho} \leq \hat{\lambda}\rho$, the developer chooses $\alpha^*(\hat{\lambda}, \rho) = 0$.

If the patentee approaches, the parties engage in a Nash-bargaining. Technology B is always implemented and the developer pays a license L, which depends on ρ, β, $\bar{\rho}$, the R&D cost function $c(\cdot)$. We simply denote it $L(\rho)$.

In any equilibrium, the developer's belief must be consistent with Bayes' rule so

$$\hat{\lambda}(\phi^*) = \frac{(1-\phi^*)\lambda}{(1-\phi^*)\lambda + 1 - \lambda}.$$

Efficient Equilibrium ($\phi^* = 1$). For efficiency, the patentee must approach with probability 1. Conditional on not being approached (off-path if there is a patentee), the developer updates his belief to $\hat{\lambda} = 0$ and invests $\alpha^*(0, \rho)$. This equilibrium exists whenever

$$\Pi_{NA}(\rho, \alpha^*(0, \rho)) \leq L(\rho).$$

Always Lurking Equilibrium ($\phi^* = 0$). If the patentee never approaches, the developer updates his belief to $\hat{\lambda} = \lambda$ and invests $\alpha^*(\lambda, \rho)$. For this to be an equilibrium we need

$$\Pi_{NA}(\rho, \alpha^*(\lambda, \rho)) \geq L(\rho).$$

Mixed-Strategy Equilibrium. If the patentee approaches with probability $\phi^* \in (0, 1)$, the developer updates his belief to $\hat{\lambda}(\phi^*)$ and invests $\alpha^*(\hat{\lambda}(\phi^*), \rho)$. For this to be an equilibrium, the patentee must be indifferent between approaching or not, so we need

$$\Pi_{NA}(\rho, \alpha^*(\hat{\lambda}(\phi^*), \rho)) = L(\rho),$$

where $\hat{\lambda}(\phi^*)$ is given by Bayes' rule.

A.1 Fixed Development Cost

In the baseline model, the developer is able to adopt technology B with probability 1 and without paying any costs. In that case, patents do not contribute any social value. In practice, patents are valuable precisely because they avoid duplication costs and reduce failure to develop new technologies. It is easy to imagine situations where different technology adoption choices carry both different probabilities of success and different costs. To capture this in a simple way, let a developer who attempts to adopt technology B pay a cost $K\pi$ (where $K > 0$) for the effort and succeed with probability $\alpha \in (0, 1)$. Technology A remains costless and failure-free when adopted. Hence, a developer that tries but fails to adopt B will adopt A. Under a belief $\hat{\lambda}$ that there is an infringement threat, the developer finds worth paying the fixed cost when $\alpha(1 - \hat{\lambda}\rho)\pi + (1 - \alpha)(1 - \bar{\rho})\pi - \pi K > (1 - \bar{\rho})\pi$ or, equivalently,

$$\bar{\rho} - \hat{\lambda}\rho \geq \frac{K}{\alpha}. \tag{8}$$

If the patentee approaches ex ante, the parties Nash-bargain under complete information. We have:

Lemma A.1. *If the patentee approaches ex ante, the Nash-barganing payoffs are*

$$\Pi_P^{NB} = \begin{cases} (\alpha\rho + \beta(\bar{\rho}(1 - \alpha) + K))\pi & \text{if } \rho \leq \bar{\rho} - K/\alpha \\ \beta\bar{\rho}\pi & \text{if } \rho > \bar{\rho} - K/\alpha \end{cases}$$

$$\Pi_D^{NB} = \begin{cases} (1 - \alpha\rho - \beta(\bar{\rho}(1 - \alpha) + K))\pi & \text{if } \rho \leq \bar{\rho} - K/\alpha \\ (1 - \beta\bar{\rho})\pi & \text{if } \rho > \bar{\rho} - K/\alpha \end{cases}.$$

In the benchmark case of $\lambda = 1$, there is always an equilibrium bargain. The patentee is no longer indifferent when the developer ignores patents because there is a gain to be made by avoiding duplication and preventing failure.

Proposition A.1. *If $\lambda = 1$, the patentee always approaches for sure and the developer chooses B in equilibrium.*

Next, consider the case of an uncertain patentee ($\lambda < 1$).

Proposition A.2. *If $K \geq \alpha\bar{\rho}$ the developer always chooses A if not approached. Thus, the patentee always approaches. If $K < \alpha\bar{\rho}$, there are three non-generic cases depending on ρ:*

1. *When $\rho \leq \max\{\frac{\beta\bar{\rho}}{\alpha}, \bar{\rho} - \frac{K}{\alpha}\}$, there is a unique equilibrium in which the developer chooses B and the patentee approaches (ex ante patent transfer)*
2. *When $\rho \in \left(\max\{\frac{\beta\bar{\rho}}{\alpha}, \bar{\rho} - \frac{K}{\alpha}\}, \max\{\frac{\beta\bar{\rho}}{\alpha}, \frac{\bar{\rho}-\frac{K}{\alpha}}{\lambda}\}\right)$, there is a unique equilibrium in which the developer chooses B and the patentee lurks (ex post patent transfer).*
3. *When $\rho > \max\{\frac{\beta\bar{\rho}}{\alpha}, \frac{\bar{\rho}-\frac{K}{\alpha}}{\lambda}\}$ there is a unique equilibrium in which the developer chooses B with probability $1 - a^* = \frac{\beta\bar{\rho}}{\alpha\rho}$ and the patentee lurks with probability $1 - \phi^* = \frac{1-\lambda}{\lambda}\left(\frac{\bar{\rho}-\frac{K}{\alpha}}{\rho-\left(\bar{\rho}-\frac{K}{\alpha}\right)}\right)$.*

Figure 7: Costly and probabilistic adoption

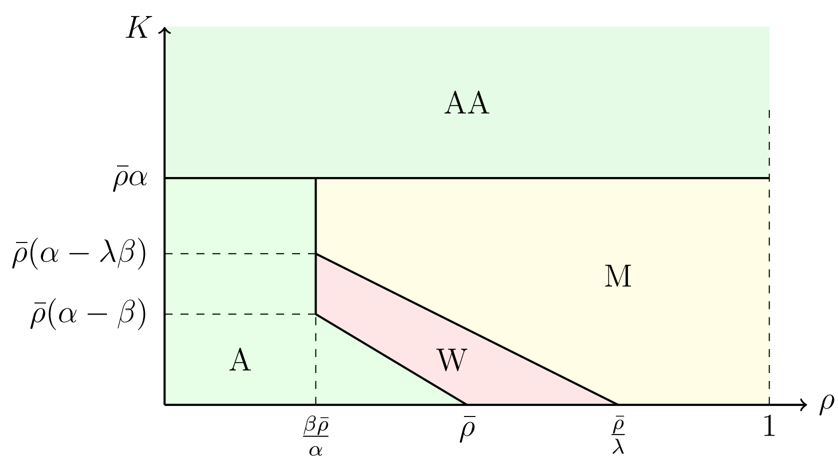

Figure 7 summarizes the cases. Equilibrium for area AA is when $K \geq \alpha\bar{\rho}$; even knowing there is no patentee, the developer does not want to choose B because it is too costly. In equilibrium, the patentee always approaches and extract the surplus associated with the incremental value of B. Part 1 of Proposition A.2 corresponds to Area A. In this region, the developer would choose B is not approached. The patentee approaches and extracts the surplus associated with avoiding duplication costs and preventing failure. Part 2 of Proposition A.2 corresponds to Area W. In this region, the developer chooses B if not approached. The patentee sacrifices the rents associated with avoiding duplication costs and preventing failure in order to not give up its informational advantage and hold up the developer. Lastly, part 3 of Proposition A.2

corresponds to Area M. In this region, there is mixing: the patentee mixes between lurking and approaching, while the developer mixes between trying B or adopting A.

Note that the baseline model ($K = 0$ and $\alpha = 1$) correponds to the x-axis in Figure 7. For $K < \bar{\rho}(\alpha - \beta)$, the equilibrium where the patentee approaches for sure is now unique because the surplus extraction associated with avoiding duplication costs and preventing failure breaks the indifference of the patentee in the baseline model. Next, as K increases from 0 to $\bar{\rho}(\alpha - \beta)$, the cutoffs separating the regions A, W, and M shift to the left. The developer must pay K in attempting to adopt B and may fail. Hence, they are more inclined to adopt A, both as an outside option in the case where the patentee has approached and as a pure adoption choice in the case where the patentee has not approached, which shrinks the A and W, and expands the region M.

When $0 < K < \bar{\rho}(\alpha - \beta)$, we can have inefficient adoption in equilibrium for values of ρ where adoption was efficient in the baseline case. For example, inside the region W, adoption may fail because $\alpha < 1$. In region M there are two effects. Now there is mixing, so there is inefficient adoption when the developer chooses A and the patentee does not approach. But there is also inefficiency when the developer attempts B and fails and the patentee does not approach. As K increases or α decreases, the patentee approaches more often. However, as α decreases the developer chooses A less often if not approached. Essentially, it attempts B more often because failure is more common; to get the same frequency of success requires more frequent attempts.

Proof of Lemma A.1

Proof. **Approach.** If the patentee approaches, the developer can pay a fee T to use the patentee's knowledge/patent to implement B and succeed with probability 1 without the need to pay the cost K. In that case, the patentee gets T and the developer gets $\pi - T$. If the deal is rejected, the developer will hold a belief $\hat{\lambda} = 1$ and not invest to develop B if $\bar{\rho} < \rho + K/\alpha$. In that case, payoffs are 0 for the patentee and $(1 - \bar{\rho})\pi$ for the developer. If $\bar{\rho} \geq \rho + K/\alpha$, the developer will invest to develop B and payoffs are $\alpha\rho\pi$ for the patentee and $(\alpha(1-\rho) + (1-\alpha)(1-\bar{\rho}) - K)\pi$ or, equivalently, $(1 - \bar{\rho} + \alpha(\bar{\rho} - \rho) - K)\pi$ for the developer.

Case 1. If $\bar{\rho} < \rho + K/\alpha$, the joint payoff from agreement is π while the disagreement payoff is $(1 - \bar{\rho})\pi$. Thus, the equilibrium transfer is $T = \beta\bar{\rho}\pi$. (This is the same as in the baseline case). The patentee extract rents for *encouraging efficient adoption.*

Case 2. If $\bar{\rho} \geq \rho + K/\alpha$, the joint payoff from agreement is π. Joint disagreement payoffs are $(1 - \bar{\rho}(1-\alpha) - K)\pi$. Thus, the equilibrium transfer is $T = \alpha\rho\pi + \beta(\bar{\rho}(1-\alpha) + K)\pi$. The baseline case obtains when $\alpha = 1$ and $K = 0$. In contrast to the baseline case, the patentee is able to extract rents from his "know-how," which *avoids costly duplication effort and prevents failure to adopt B.* □

Proof of Proposition A.2

Proof. 1. $\phi^* = 1$. In this case, $\hat{\lambda} = 0$, so (8) becomes $\bar{\rho} \geq K/\alpha$.

(a) If $\bar{\rho} \geq K/\alpha$, the developer invests to adopt B if not approached.

i. If $\bar{\rho} < \rho + K/\alpha$, the patentee gets $\alpha\rho\pi$ by not approaching and $\beta\bar{\rho}\pi$ by approaching. Approaching reveals the existence of the patentee which changes the decision to invest when the negotiation breaks down. Thus, approaching is optimal when $\alpha\rho \leq \beta\bar{\rho}$. This equilibrium exists when

$$\rho \in \left(\bar{\rho} - \frac{K}{\alpha}, \frac{\beta\bar{\rho}}{\alpha}\right].$$

ii. If $\bar{\rho} \geq \rho + K/\alpha$ it is optimal to approach to capture rents from avoiding costly duplication effort and also to prevent failure in the adoption, which happens with probability $1 - \alpha$. The additional surplus of approaching is strictly positive, $\beta(\bar{\rho}(1-\alpha) + K)$.

(b) If $\bar{\rho} < K/\alpha$, the developer never invests to adopt B if not approached. It is optimal to approach because the patentee gets 0 by not approaching, and a strictly positive payoff from approaching.

2. $\phi^* = 0$. In this case, $\hat{\lambda} = \lambda$, so (8) becomes $\frac{\bar{\rho} - K/\alpha}{\lambda} \geq \rho$.

(a) When $\frac{\bar{\rho} - K/\alpha}{\lambda} \geq \rho$, the developer invests to develop B if not approached.

i. If $\bar{\rho} < \rho + K/\alpha$, the patentee gets $\alpha\rho\pi$ by not approaching and $\beta\bar{\rho}\pi$ by approaching. Approaching reveals the existence of the patentee which changes the decision to invest when the negotiation breaks down. Thus, not approaching is optimal when $\alpha\rho \geq \beta\bar{\rho}$. This equilibrium exists when

$$\rho \in \left(\max\left\{\bar{\rho} - \frac{K}{\alpha}, \frac{\beta\bar{\rho}}{\alpha}\right\}, \frac{\bar{\rho} - \frac{K}{\alpha}}{\lambda}\right].$$

ii. If $\bar{\rho} \geq \rho + K/\alpha$ it is optimal to approach to capture rents from avoiding costly duplication effort and preventing failure to discover B. Thus, there is no equilibrium with $\phi^* = 0$ in this region.

(b) When $\frac{\bar{\rho}-K/\alpha}{\lambda} < \rho$, the developer never invests in developing B if not approached. It is optimal to approach because the patentee gets 0 by not approaching, so an equilibrium where the patentee never approaches is not an equilibrium in this region.

3. $\phi^* \in (0,1)$. Suppose the developer is indifferent between A and invest to develop B, and chooses A with probability a. For the developer to be indifferent we need the patentee to approach with probability ϕ such that

$$\bar{\rho} - \hat{\lambda}(\phi)\rho = \frac{K}{\alpha}.$$

Since $\hat{\lambda}(\phi) \in [0, \lambda]$, we need $\bar{\rho} \geq K/\alpha$. If this holds, then it is possible to meet the inequality above when

$$\frac{\bar{\rho} - K/\alpha}{\lambda} \leq \rho.$$

Note that this condition implies $\bar{\rho} < \alpha\rho + K/\alpha$ meaning that the developer chooses A if approached but the negotiation breaks down. Thus, the patentee gets $\beta\bar{\rho}\pi$ by approaching. By not approaching, the developer tries to develop B first with probability $(1-a)$, so the patentee gets $(1-a)\alpha\rho\pi$. Thus, the patentee is indifferent between approaching and not when

$$(1-a)\alpha\rho = \beta\bar{\rho}.$$

It is possible to find a to satisfy this condition when $\rho \geq \frac{\beta\bar{\rho}}{\alpha}$. Thus, there exists a mixed equilibrium when

$$\rho \geq \max\left\{\frac{\beta\bar{\rho}}{\alpha}, \frac{\bar{\rho} - K/\alpha}{\lambda}\right\}$$

If the mixed equilibrium exists, the mixing probabilities are

$$a^* = 1 - \frac{\beta\bar{\rho}}{\alpha\rho}, \quad \hat{\lambda}(\phi^*) = \frac{\bar{\rho} - K/\alpha}{\rho},$$

□